

Superconducting States in pseudo-Landau Levels of Strained Graphene

Bruno Uchoa¹ and Yafis Barlas²

¹ *Department of Physics and Astronomy, University of Oklahoma, Norman, OK 73069, USA and*

² *Department of Physics and Astronomy, University of California at Riverside, Riverside, CA 92521, USA*

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We describe the formation of superconducting states in graphene in the presence of pseudo-Landau levels induced by strain, when time reversal symmetry is preserved. Due to the incompressibilities associated with completely filled pseudo-Landau levels, superconductivity in strained graphene is quantum critical at integer filling factors, whereas at partial fillings superconductivity survives at weak coupling. In this limit, the critical temperature scales *linearly* with the coupling strength and can be orders of magnitude larger than in conventional weak coupling superconductors. We argue that superconductivity can be induced by electron-phonon coupling and that the transition temperature can be controlled with the amount of strain and with the filling fraction of the Landau levels.

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Graphene is a single atomic sheet of carbon with electronic excitations that behave as massless Dirac quasiparticles [1, 2]. In general, graphene seems to be insensitive to electronic many body instabilities [3], except in the quantum Hall regime [4], where fractional quantum Hall states [5–7] have been observed. We claim that one promising route to induce intrinsic superconductivity in graphene is to reconstruct the electronic density of states (DOS) into a discrete spectrum of Landau levels (LLs) with the application of strong strain fields. Strain configurations in a specific engineered form [8] mimic the application of strong uniform magnetic fields that can be as large as 300T [9], but preserve time reversal symmetry (TRS). Quantum Hall states induced by pseudomagnetic fields have been conjectured to give rise to topological order in strained graphene with spontaneously broken TRS [10, 11].

In this letter, we describe the formation of intrasublattice TRS spin singlet states which occupy the LLs produced by elastic deformations in graphene. Since the overall wavefunction is anti-symmetric, the spin singlet wavefunctions are even under valley exchange, and are robust against backscattering [12], unlike the corresponding triplet states, which break inversion symmetry [10]. We show that at integer filling factors where the normal state becomes incompressible due to Pauli blocking, the superconducting order parameter has a quantum critical point at the mean field level, $\Delta \propto |x - x_c|^{1/2}$, where x_c is a LL dependent critical coupling, in contrast with the unstrained case, which is quantum critical only at the neutrality point [13–15]. Near complete filling of the LLs, a sequence of quantum critical points can be experimentally accessed by controlling the filling factor in the *weak* coupling regime, $x \lesssim x_c$. At partial filling of the LLs, we show that the zero temperature gap $\Delta \propto x$ has a *linear* scaling with coupling and strain in the weak coupling limit $x \ll x_c$, and can be made orders of magnitude larger than in conventional weak coupling superconductors, where $\Delta \propto e^{-1/x}$. We identify experimental signa-

tures for this state, and we propose that in the presence of substrates that screen Coulomb interactions at length scales larger than the magnetic length, superconductivity can be triggered by conventional electron-phonon coupling.

In the continuum description of the problem, the low energy Hamiltonian of strained graphene is

$$\mathcal{H}_0 = \sum_{\sigma, \alpha} \int d\mathbf{x} \Psi_{\sigma, \alpha}^\dagger(\mathbf{x}) [v(-i\nabla + \alpha \mathbf{A}_s) \cdot \vec{\sigma}_\alpha - \mu] \Psi_{\sigma, \alpha}(\mathbf{x}) \quad (1)$$

where $\sigma = \uparrow, \downarrow$ is the spin index, $\alpha = \pm$ indexes the two valleys, $\vec{\sigma}_\alpha = (\alpha\sigma_x, \sigma_y)$ is a vector of Pauli matrices, μ is the chemical potential away from half filling, $v = 6\text{eV}\text{\AA}$ is the Fermi velocity, $\Psi_\sigma = (\psi_{a,+}, \psi_{b,+}, \psi_{a,-}, \psi_{b,-})_\sigma$ is a 4 component spinor in the (a, b) sublattice pseudospin and in the two valleys and \mathbf{A}_s is the pseudo vector potential, which couples to the electrons as a magnetic field pointing in opposite directions in the two different valleys, preserving time reversal invariance. From now on, we assume strain configurations which produce uniform pseudo-magnetic fields, B_s [8].

In the Landau gauge, where $\mathbf{A}_s = (-B_s y, 0)$, and B_s is the pseudomagnetic field, the electronic wavefunction takes the form $\Psi_{k, \sigma}(x, y) = \exp(ikx) \Theta_\sigma(y)$, where $\Theta_\sigma(y)$ is the eigenspinor of a 1D Hamiltonian. This Hamiltonian can be expressed in terms of ladder operators of the 1D harmonic oscillator, $a \equiv (\xi + \partial_\xi)/\sqrt{2}$, $a^\dagger \equiv (\xi - \partial_\xi)/\sqrt{2}$, where $\xi \equiv \ell_B k - y/\ell_B$ is a dimensionless variable related to the valley dependent guiding center $X = -k\ell_B^2$, with $\ell_B = \sqrt{\hbar/eB_s}$ (restoring \hbar) the effective magnetic length, and e the electron charge. In what follows, we define the valley dependent operator $\hat{D}(\xi)$,

$$v(-i\nabla + \mathbf{A}_s) \cdot \vec{\sigma} = \sqrt{2} \frac{v}{\ell_B} \begin{pmatrix} 0 & a \\ a^\dagger & 0 \end{pmatrix} \equiv \hat{D}(\xi), \quad (2)$$

which takes the form $-v(-i\nabla - \mathbf{A}_s) \cdot \vec{\sigma} = -\hat{D}(\bar{\xi})$ in the opposite valley, with $\bar{\xi} = \ell_B k + y/\ell_B$.

In the presence of an effective attractive potential U that stabilizes the superconducting state, the Bogoliubov-deGennes (BdG) Hamiltonian is $\hat{\mathcal{H}}_{\text{BG}} = \int d\mathbf{x} \Phi^\dagger(\mathbf{x}) \hat{\mathcal{H}}_{\text{BG}} \Phi(\mathbf{x})$,

$$\hat{\mathcal{H}}_{\text{BG}} = \begin{pmatrix} \hat{\mathcal{H}}_0(\mathbf{A}_s) & \hat{\Delta} \\ \hat{\Delta}^* & -\mathcal{T}\hat{\mathcal{H}}_0(\mathbf{A}_s)\mathcal{T}^{-1} \end{pmatrix}, \quad (3)$$

where $\hat{\mathcal{H}}(\mathbf{A}_s) = \hat{\mathcal{D}}(\xi) \otimes \nu^+ - \hat{\mathcal{D}}(\xi) \otimes \nu^- - \mu\sigma_0\nu_0$, is the normal state Hamiltonian of strained graphene written in valley and sublattice spaces, $\nu^\pm = (\nu_0 \pm \nu_z)/2$ are projectors in the \pm valley spaces, with Pauli matrices ν_i ($i = x, y, z$), and $\Phi = (\Psi_{k,\uparrow}, \Psi_{-k,\downarrow}^\dagger)$ is the 8 component spinor in the Nambu space, with Pauli matrices τ_i ($i = x, y, z$). The off diagonal term, $\hat{\Delta}$, is a pairing matrix that describes the formation of Cooper pairs $\Delta^2 = U \text{tr} \langle \Psi_{k,\sigma}^\dagger \hat{\Delta} \Psi_{-k,-\sigma} \rangle$. In strained graphene, the time reversal symmetry operation $\mathcal{T}\hat{\mathcal{H}}_0(\mathbf{A}_s)\mathcal{T}^{-1}$ leaves the Hamiltonian invariant under an additional exchange between valleys, in contrast with the case of conventional magnetic fields, which explicitly break TRS [16].

In the intra-sublattice s -wave pairing state, which corresponds to the pairing matrix $\hat{\Delta} = \Delta\sigma_0\nu_x$, the eigenvector problem $\hat{\mathcal{H}}_{\text{BG}}\Phi(x, \xi) = E\Phi(x, \xi)$ can be solved by decomposing Hamiltonian (3) into two equivalent copies of 4×4 BdG Hamiltonians in pseudospin and Nambu spaces. In the reduced Nambu basis $\bar{\Phi} = (\Psi_{\uparrow,+}, \Psi_{\downarrow,-}^\dagger)$,

$$\bar{\mathcal{H}}_{\text{BG}} = \begin{pmatrix} \hat{\mathcal{D}}(\xi) - \mu & \Delta \\ \Delta^* & -\hat{\mathcal{D}}(\xi) + \mu \end{pmatrix}. \quad (4)$$

Fixing the gauge of the gap Δ to be real, the eigenvalue problem $(E - \bar{\mathcal{H}}_{\text{BG}})\bar{\Phi} = 0$ is equivalent to

$$\mathcal{M}\bar{\Phi} \equiv (E + \bar{\mathcal{H}}'_{\text{BG}})(E - \bar{\mathcal{H}}_{\text{BG}})\bar{\Phi} = 0$$

where $\hat{\mathcal{H}}'_{\text{BG}} \equiv \mathcal{C}\bar{\mathcal{H}}_{\text{BG}}\mathcal{C}^{-1} = (\hat{\mathcal{D}} + \mu) \otimes \tau_3 + \Delta\tau_1$ is the charge conjugated BdG Hamiltonian ($\mu \rightarrow -\mu$). When the matrix \mathcal{M} is applied in a Nambu state $\bar{\Phi}$ where $\Psi_{\sigma,\alpha}^{(N)} = (\psi_{|N|-1}, s\psi_{|N|})_{\sigma,\alpha}$, with N the index of the Landau levels, and $s(N) \equiv \text{sgn}(N)$ accounts for the two branches of LLs in the conduction and valence bands, \mathcal{M} can be cast in the form $\mathcal{M}\bar{\Phi} = (E + \mathcal{H}_+)(E - \mathcal{H}_-)\bar{\Phi}$, with $\mathcal{H}_\pm = (s\omega_c\sqrt{|N|} \pm \mu)\tau_3 + \Delta\tau_1$, and $\omega_c = \sqrt{2}v/\ell_B$. \mathcal{H}_- is equivalent to Hamiltonian (4) and gives the energy spectrum

$$\pm E_N = \pm[(s\omega_c\sqrt{|N|} - \mu)^2 + \Delta^2]^{1/2}, \quad (5)$$

with eigenstates given by

$$\Psi_{\pm,\sigma,\alpha}^{(N)}(\xi) = \beta_{\pm,\sigma,\alpha}^N \begin{pmatrix} \phi_{|N|-1}(\xi) \\ s\phi_{|N|}(\xi) \end{pmatrix} e^{ikx}, \quad (6)$$

for the states $(\sigma, \alpha) = (\uparrow, -)$ and $(\downarrow, +)$, where $\beta_{\pm,\uparrow,-}^N = 1/\sqrt{2}[1 \pm (s\omega_c\sqrt{|N|} - \mu)/E_N]^{1/2}$, and $\beta_{\pm,\downarrow,+}^N =$

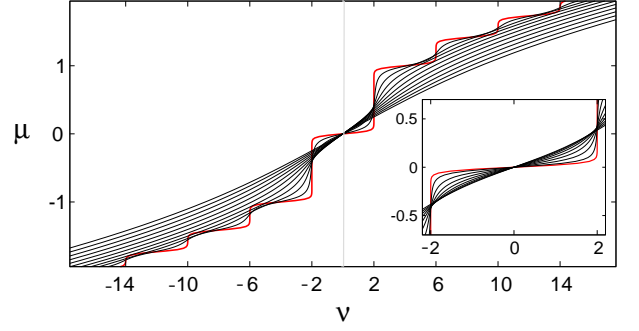


Figure 1: Dependence of the chemical potential μ in units of ω_c with the filling factor ν . Red (light) curve: normal state $\Delta = 0$. Solid black curves: superconducting state, with Δ/ω_c ranging from 0 (red line) to 1. All curves are plotted at $T/\omega_c = 0.02$. Inset: detail of the zero LL ($n = 0$). The red curve is analytically described by Eq. (10), while solid ones at $\Delta/\omega_c \ll 1$ are described by Eq. (9) in the $T \rightarrow 0$ limit.

$\mp 1/\sqrt{2}[1 \mp (s\omega_c\sqrt{|N|} - \mu)/E_N]^{1/2}$. $\phi_N(\xi)$ denotes conventional LL wavefunctions, with $\phi_{-1}(\xi) = 0$. In the zero LL, the Cooper pairs occupy only one sublattice, explicitly breaking the \mathbb{Z}_2 sublattice symmetry of graphene. As anticipated, the BdG quasiparticle spectrum is discrete and can be indexed by the LL index N . The superconducting ground state is given by [17]

$$|\Psi_0\rangle = \prod_{N,X} \left(u_N + v_N c_{N,X,\uparrow,-}^\dagger - c_{N,-X,\downarrow,+}^\dagger \right) |0\rangle, \quad (7)$$

where $u_N = \beta_{+,\uparrow,-}^N$, $v_N = -\beta_{+,\downarrow,+}^N$ and $c_{N,X,\sigma,\alpha}^\dagger$ are fermionic creation operators of the relativistic LLs. This wavefunction describes intrasublattice pairing across opposite valleys, within the same LL.

The discontinuous behavior of the chemical potential with the pseudomagnetic field can be calculated by fixing the total number of particles in the system. Although the ground state wave function does not conserve the number of particles, the distribution is sharply peaked around the average in the thermodynamic limit [18], $\mathcal{N} = gN_\phi \sum_{N=-\infty}^{\infty} [u_N^2 f(E_N) + v_N^2 f(-E_N)]$, where $f(E) = (1 + e^{E/T})^{-1}$ is the Fermi distribution, $g = 4$ is the valley and spin degeneracy, and $N_\phi = A/(2\pi\ell_B^2)$ is the number of flux quanta for a total area A , which sets the LLs degeneracy. In the low temperature and weak coupling regime $T, \Delta \ll v/\ell_B$, where the deep energy states $N < n$ are fully occupied, with n the highest occupied LL, the constraint becomes

$$2(\nu/g - n) = -[(s\omega_c\sqrt{|n|} - \mu)/E_n] \tanh[E_n/(2T)], \quad (8)$$

where $\nu = \mathcal{N}/N_\phi - g(N_\Lambda + 1/2)$ is the filling factor, and $N_\Lambda = (D/\omega_c)^2 > 0$ is an ultraviolet cutoff that regularizes the number of negative energy states, where $D \sim 6\text{eV}$ is the bandwidth. In particular, at $T = 0$, the chemical

potential

$$\mu(0, \nu) = s\omega_c \sqrt{|n|} + \frac{\Delta(\nu - gn)}{\sqrt{[g(n + \frac{1}{2}) - \nu][\nu - g(n - \frac{1}{2})]}}, \quad (9)$$

remains pinned to the n -th LL when half filled ($\nu = gn$) for small Δ , and shows a power law divergence when the highest occupied LL is completely filled, at integer fillings $\nu = g(n \pm 1/2)$, indicating an incompressibility. In the opposite regime, when $T, \Delta \gtrsim v/\ell_B$, the system crosses over to the usual Fermi liquid behavior when the electrons have multiple transitions between different LL. In the normal state ($\Delta = 0$), as expected, the chemical potential

$$\mu(T, \nu) = s\omega_c \sqrt{|n|} + T \ln \left[\frac{\nu - g(n - \frac{1}{2})}{g(n + \frac{1}{2}) - \nu} \right], \quad (10)$$

is also pinned to the energy of the highest occupied LL at partial fillings $\nu = gn$, and shows a logarithmic divergence at integer fillings $\nu = g(n \pm 1/2)$, when the corresponding LL becomes incompressible. When all LLs are taken into account, as shown in the red (light) curve in Fig. 1, at integer fillings $\nu_I(n) = g(n + \frac{1}{2})$, the chemical potential for the normal state does not diverge but sits half way between the LLs in the zero temperature limit, $h(n) \equiv \mu[0, \nu_I(n)]/\omega_c = (\sqrt{|n+1|} + \sqrt{|n|})/2$, unlike the $T \rightarrow 0$ limit at partial filling, where $\mu(0, \nu) = s\omega_c \sqrt{|n|}$.

In Fig. 1, we compute numerically the contribution of all LLs to the chemical potential at fixed temperature for different values of Δ . At $\nu = 0$, the system is half filled and the chemical potential is always pinned to the energy of the zero LL. When $\Delta \sim v/\ell_B$, the chemical potential $\mu(\nu)$ detaches from the discrete profile of LL plateaus, and becomes a smooth function. Eq. (9) and (10) describe analytically the numerical curves shown in the inset of Fig. 1 when $\Delta/\omega_c \ll 1$.

At the mean field level, the free energy of the superconducting state is $F = -TgN_\phi \sum_{\gamma=\pm} \sum_{N=-\infty}^{\infty} \ln(1 + e^{-\gamma E_N/T}) - \bar{A}|\Delta|^2/U$, where \bar{A} is the total area normalized by the size of the unit cell. Minimization of the free energy gives the gap equation

$$1 = -(U/2)g\bar{N}_\phi \sum_{N=-\infty}^{\infty} \tanh[E_N(T, \nu)/(2T)]/E_N, \quad (11)$$

where $\bar{N}_\phi = 3\sqrt{3}a^2/(4\pi\ell_B^2)$ is the number of flux quanta per unit cell, with $a = 1.42\text{\AA}$ the lattice spacing. Defining $x \equiv |U|g\bar{N}_\phi/\omega_c \propto |U|/\ell_B$ as the dimensionless coupling parameter that controls the strength of interactions and strain, at half filling ($\nu = 0$), the zero temperature gap in the weak coupling regime $T_c \ll v/\ell_B$ is

$$\Delta^{(0)}(0) = [v x / (\sqrt{2}\ell_B)] / [1 - \zeta_A(1/2)x], \quad (12)$$

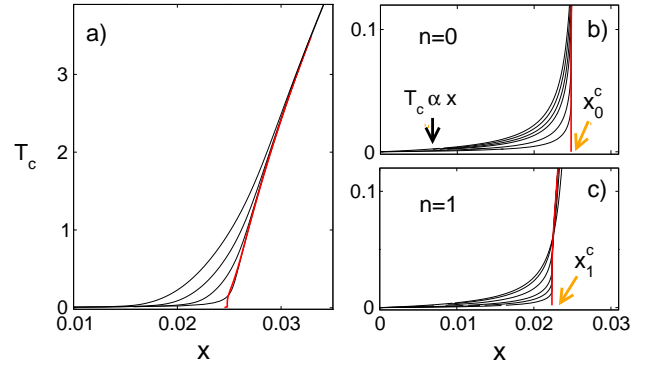


Figure 2: Critical temperature T_c/ω_c vs. the dimensionless coupling strength $x \equiv |U|gN_\phi\ell_B/(\sqrt{2}v)$. a) red (light) curve: $\nu = 2$, where the transition is quantum critical below $x_0^c \approx 0.025$. Solid black curves: $\nu = 0, 8, 24$ and $\nu = 40$, from right to left. b) $\nu = 0, 1.2, 1.6, 1.98, 1.9998$ and 2 , from left to right. c) $\nu = 4, 5.2, 5.6, 5.98, 5.9998$ and 6 , from left to right. At partial filling of the LL, $T_c \propto x$ in the $x \rightarrow 0$ limit.

where $\zeta_A(\frac{1}{2}) = \sum_{N=1}^{N_\Lambda} 1/\sqrt{N}$ is the zeta function regularized by an ultraviolet cut-off. The ratio between the critical temperature and the zero temperature gap at half filling is a universal number, $2T_c = \Delta^{(0)}(0)$. In the weak coupling limit, $T_c \sim \sqrt{2}vx/\ell_B \propto B_s$ has a linear scaling with the coupling and with the amount of strain [19]. This scaling contrasts with the case of conventional weak coupling superconductors, where $T_c \propto \exp(-1/x)$ decreases exponentially with the effective coupling. As the coupling x becomes larger, the system eventually crosses over to the strong coupling regime, when $T_c \gtrsim v/\ell_B$, as shown in Fig. 2. In the critical regime, when $\Delta/T_c \ll 1$, the gap at $\nu = 0$ is given by

$$\Delta^{(0)}(T) = 2^{\frac{1}{4}}(v/\ell_B)^{\frac{3}{2}} \sqrt{1 - T/T_c} / [\zeta(3/2) T_c]^{\frac{1}{2}}, \quad (13)$$

at weak coupling, and scales with $\Delta^{(0)} \propto B_s^{3/4}$, where $\zeta(3/2) \approx 2.61$ is a zeta function. In Fig. 2a, we show the dependence of the critical temperature with the coupling x for different filling factors. The red curve is the phase transition for $\nu = 2$, which is *quantum critical*.

At integer filling factors $\nu_I(n) = g(n + \frac{1}{2})$, the normal state becomes incompressible at temperatures much smaller than the LL spacing due to Pauli blocking, inhibiting charge density fluctuations required to preserve the electromagnetic gauge invariance in a superconductor[20]. In that case, the phase transition becomes quantum critical, and the emergence of superconductivity requires a critical coupling to overcome the electronic incompressibility of the normal state. At those integer fillings, the zero temperature gap is

$$\Delta^{(\nu_I)}(0) = 2\sqrt{2\gamma_n}(v/\ell_B)\sqrt{x/x_n^c - 1}, \quad (14)$$

where

$$x_n^c = 2 / \sum_{N=-N_\Lambda}^{N_\Lambda} |s\sqrt{|N|} - h(n)|^{-1} \quad (15)$$

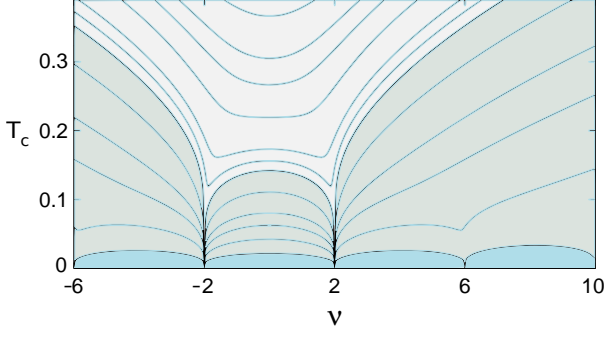


Figure 3: Phase diagram T_c/ω_c as a function of the filling factor ν for different coupling strengths, x (solid curves). Blue (dark) region: $x < x_1^c \approx 0.022$. Grey region: $x_1^c < x < x_0^c \approx 0.025$. Light region: $x > x_0^c$. The critical temperature drops to zero at integer filling factors $\nu_I(n) = g(n + \frac{1}{2})$ whenever $x < x_n^c$, where x_n^c is the critical coupling of the $\nu_I(n)$ state.

is the quantum critical coupling of the $\nu = g(n + \frac{1}{2})$ state, when the n -th LL is completely filled, $\gamma_n^{-1} = \sum_{N=-\infty}^{\infty} |s\sqrt{|N|} - h(n)|^{-3}$ is a constant and $h(n)$ was defined below Eq. (10). For a magnetic length of $\ell_B \sim 30\text{\AA}$, which corresponds to a pseudomagnetic field $B_s \sim 100\text{T}$, we have $N_\Lambda \sim 400$. At $n = 0$, the critical coupling is $x_0^c \approx 0.025$, as depicted by the arrow in Fig. 2b for the red curve, crossing over to a smooth transition for partial filling factors, as shown in the black curves in the same panel, where $0 \leq \nu < 2$. For $n = 1$, the critical coupling drops to $x_1^c \approx 0.022$ (Fig. 2c), with the solid curves in the same panel indicating a crossover to partial filling factors in the range $4 \leq \nu < 6$. For higher LL, x_n^c decreases further as n becomes large.

For fixed coupling strength, the critical temperature evolves with ν as a series of lobes and drops to zero at integer filling factors $\nu_I(n) = g(n + \frac{1}{2})$, whenever $x < x_n^c$, as shown in Fig. 3. The solid blue area in Fig. 3 depicts the region $x < x_1^c$, which is quantum critical for the first two Landau levels, while the gray region $x_1^c < x < x_0^c$ is quantum critical in the zero LL only. When x grows larger than x_n^c , the system undergoes a quantum phase transition in the state $\nu = \nu_I(n)$, when superconductivity emerges at that filling factor. For $\nu \sim \nu_I(n)$, the critical temperature scales as

$$T_c(x, \nu) \propto |\nu - \nu_I(n)|^\delta \quad (16)$$

for $x < x_n^c$, with possible logarithmic behavior, where $\delta \sim 0.2(3)$ is the exponent numerically extracted for the $\nu = \pm 2$ states. This behavior may lead to the experimental observation of quantum criticality in graphene by controlling the filling factor of the LL in the *weak* coupling limit $x \ll x_n^c$. In two dimensions, the critical temperature sets the onset of Cooper pair formation, while long range phase coherence is lost above the Kosterlitz-Thouless transition temperature $T_{KT} < T_c$ [21], where pairs of vortices and anti-vortices unbind. Those effects

will likely be relevant for transport and will be discussed elsewhere.

Besides transport measurements in *long* graphene junctions, where a supercurrent is expected to flow along the edges [22], one experimental signature of superconductivity in strained graphene is the specific heat at fixed volume, $C_V = -T(\partial^2 F / \partial T^2)_V$. At low temperature, $T \ll T_c \ll v/\ell_B$, the specific heat of the $\nu = 0$ state is $C_V(T) = 2gN_\phi \Delta^2(0)e^{-\Delta(0)/T}/T^2$. In the quantum limit, $\omega_c/T_c \gg 1$, the specific heat jump at the phase transition normalized by the specific heat in the normal side for $\nu = 0$ is $\Delta C/C_n = \omega_c e^{\omega_c/T_c} / [16\zeta(3/2)T_c]$ which is *non-universal*. In the weak coupling regime $x \ll x_0^c$, where $T_c \sim x\omega_c/4$,

$$(\Delta C/C_n)(x) = e^{4/x} / [4x \zeta(3/2)] \quad (17)$$

becomes exponentially large as T_c drops to zero. This feature is a signature of this state, and contrasts both with the specific heat jump expected for Dirac fermion superconductivity in unstrained graphene at half filling ($\Delta C/C_n \approx 0.35$) [13] and in weak coupling superconductors in general ($\Delta C/C_n \approx 1.43$) [18], which are universal constants.

Although strong Coulomb interactions inhibit superconductivity and can give rise to incompressible states at fractional filling factors [4], a condensate can be induced by phonons in the presence of substrates that screen the electronic repulsion at length scales larger than ℓ_B . The analysis of scanning tunneling spectroscopy experiments [23] in graphene on SiC for magnetic fields around 5T, when the LLs are well defined, indicate that the effective momentum independent electron phonon vertex in graphene is $g_0 \sim 0.1\text{eV}$ for an Einstein phonon mode at the typical frequency $\omega_{ph} \sim 0.2\text{eV}$ [23, 24]. This mode alone (E_{2g} phonon) leads to an effective attraction $U \sim -2g_0^2/\omega_{ph} \approx -0.1\text{eV}$. For a magnetic length of 20\AA , which corresponds to 2×10^{-3} flux quanta per unit cell, a net attractive coupling of that order results in a dimensionless coupling $x \sim 0.003$ and a critical temperature $T_c \sim 8\text{K}$ at the $\nu = 0$ state.

A significant enhancement of the electron-phonon coupling, and as a result T_c , can be achieved for instance by coating graphene with ionic crystals and alkaline metals such as K, which is known to form a stable crystal on top of graphene [25–27]. This mechanism can lead to measurable transition temperatures in the regime where the broadening of the highest occupied LL due to disorder effects is small compared to the level spacing. Our analysis shows that the spin singlet states are robust, and present a sequence of quantum critical points, which can be experimentally accessed by tuning the filling factor of the LLs in the weak coupling limit of the problem.

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